BUOYANCY FORCE IN MASS MEASUREMENT

new function in balances 2Y series

The publication focuses on influence of air buoyancy onto mass measurement process of materials with various density with application of electronic balances. The content presents two methods of compensating the influence of air buoyancy force. The first method uses mass standards, and the other electronic sensors. The measuring accuracy of the two methods, and differences between them are also discussed. In addition, the paper presents data on balances which feature discussed buoyancy compensation methods.

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1. Introduction

Researching new solutions on widely conceived technical thought requires multiple tests. All measuring instruments used for such research should feature adequate readability. The analysis of measurement process provides a number of factors, which used to be neglected.

This tendency is a result of common access to knowledge, which is easily transferred to practical aspects. In relation of electronic balances, it comes to making an analysis of a measurement system. The analysis should cover not only the basic parameters of an instrument, like repeatability and stability on balance indications, but also determine the influence of ambient conditions, i.e. influence of humidity and air pressure changes.

It is worth mentioning, that measurements carried out with an electronic balance, measure force which attracts the weighed object to the Earth. The relation is expressed by a formula:

$$F_G = m \cdot g$$

where: F_G – gravitational force [N]
m – object's mass [g]
g – gravitational acceleration force [~ 9,81 m/s^2]

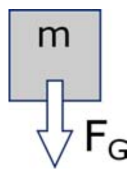


Fig. 1. Gravitational force in mass measurement process

In order to obtain a measurement result, the balance has to compensate force [F_G] which attracts the weighed object to the Earth, and then measure the compensating signal [F_C] and link it with appropriate mass. The measured signal can be either voltage, resistance, fill level signal or other value related to instrument's design. All of the activities are carried out by the manufacturer during the so called factory adjustment process. The user receives a scaled instrument – i.e. its indications are correct.

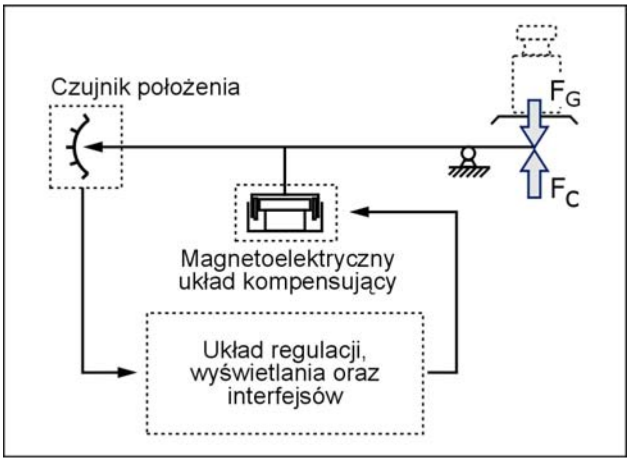


Fig. 2. Scheme of operation of an electromagnetic balance

The principle of measurement used in electronic balances, states that:

- There is a rather important dependence of the measurement result and changes in gravitational acceleration force
- During standard measurements the buoyancy force is neglected.

2. Buoyancy force

The buoyancy is a force influencing an object immersed in liquid, i.e. in liquid like water or gas, with presence of the force of gravity. The force is directed vertically and upwards, opposite to the weight. The value of buoyancy force is equal to the amount of liquid displaced by the object.

$$F_w = \rho \cdot g \cdot V$$

where:

ρ – density of a medium in which an object is placed (liquid or gas)
g – gravitational acceleration force
V – volume of displaced liquid equal to the volume of an object immersed in the liquid.

Therefore, in case of electronic balances there is the following set of forces contributing into the measurement process:

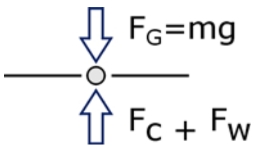


Fig. 3. Set of forces in measurement process

where:

F_G – force of gravity
F_C – force compensating the force of gravity
F_W – buoyancy force

Analysis of the set of forces provides a conclusion, that the force of gravity is a constant in a specific operation place. The compensation force is balance's reaction to the force of gravity, thus it is also a constant. The only variable, that required continuous calculating in high resolution balances is the buoyancy force.

Such a requirement arises from the fact, that buoyancy force is dependent on the density of atmospheric air at the workstation. The density depends on:

- pressure
- temperature
- humidity

While correcting an obtained measurement result by the buoyancy force, the operator obtains the actual (real) mass of a sample – similar to weighing in vacuum. Such weighing enables analyzing mass drifts in time, e.g. differential weighing eliminating errors arising from variable buoyancy force. It is especially important if a measurement is carried out with high resolution, e.g. 200 g x 10 μg ÷ 2g x 0,1 μg. In such case it is assumed, that the weighing process is carried out correctly, i.e. the smaller the sample mass, the higher balance's resolution. This procedure is applied to calibration of the mass standards of the highest accuracy class.

2.1. Evaluation of errors arising from buoyancy force

Change of air density (variable buoyancy force) during mass measurement processes may cause errors of indication. The size of the error is also related to the density of weighed sample and its mass. The balances are factory adjusted using steel mass standards with density of approximately 8000kg/m^3, and therefore:

- measurement of samples with density similar to the density of steel is almost always correct

- in case of samples with low density ranging between 500 – 4000 kg/m^3 the error arising from changes in air density may be significant. The size of the error depends on the sample mass.

In order to compensate the influence of the buoyancy force, it is necessary to determine two values. The first one is density of air, and the other one is density of tested sample.

The formula for determining air density is presented below [1]:

$$\rho_a = \frac{0,348444 \cdot p - h(0,00252 \cdot t - 0,020582)}{273,15 + t} \qquad [1]$$

where: ρ_a – air density [kg/m^3]
p – atmospheric pressure (600 hPa ≤ p ≤ 1100 hPa)
h – air humidity (20% ≤ h ≤ 80%)
t – air temperature (15^{o}C ≤ t ≤ 27 ^{o}C)

In order to determine the correct mass (corrected by the buoyancy force), the obtained weighing result has to be multiplied by coefficient, as in below formula [2] :

$$m_c = m \cdot \frac{1 - \frac{\rho_0}{\rho}}{1 - \frac{\rho_0}{\rho_c}} \qquad [2]$$

where: m_c – sample mass
ρ_0 – air density [kg/m^3]
ρ – density of standard, used to adjust the balance [8000kg/m^3]
ρ_c – density of weighed sample [kg/m^3]
m – weighing result indicated on a balance

Instance:

Atmospheric pressure 996 hPa, humidity 45 %, temperature 25 ^{o}C, sample type: leather, sample density 860 kg/m^3 and mass m_1=80 g

Calculation of air density:

$$\rho_a = \frac{0,348444 \cdot 996 - 45 \cdot (0,00252 \cdot 25 - 0,020582)}{273,15 + 25} = 1,1576 kg/m^3$$

[1] *EURAMET/cg-18v.02 Guidelines on the Calibration of Non-Automatic Weighing Instruments*
[2] *OIML D-28 "Conventional value of the result of weighing In air"*

Calculation of acutal mass m_2 of a sample:

$$m_2 = \frac{1 - \dfrac{1{,}1576\text{kg/m}^3}{8000\text{kg/m}^3}}{1 - \dfrac{1{,}1576\text{kg/m}^3}{860\text{kg/m}^3}} \cdot 80g = 80{,}096238g$$

The actual sample mass, i.e. sample mass weighed in vacuum is 80,096238g, which in practice means, that the contribution of buoyancy force in the weighing process is:

$$m_2 - m_1 = 80{,}096238 \text{ g} - 80 \text{ g} = 0{,}096238 \text{ g}$$

Sample measurement in vacuum is carried out rarely in the laboratory practice, due to insufficient measuring equipment. Therefore, it is justified to apply a factor compensating the influence of buoyancy force – this solution seems to be much simpler. In Radwag balances 2Y series the calculation is carried out automatically, and the only data requiring typing in balance’s menu is sample’s density.

2.2. Practical dependence of the factor correcting buoyancy force on the measurement result

The factor correcting the influence of buoyancy force onto mass measurement is obligatory if:

- data on actual sample mass is required. It may be necessary in cases when the same mass is weighed for multiple times in large time intervals.
- The change of sample’s mass in long period of time is monitored.

In the above cases the influence of variable air density onto the measurement result may be significant. In order to avoid such error, the user has to know the relation between variable air density and its influence on the measurement process.

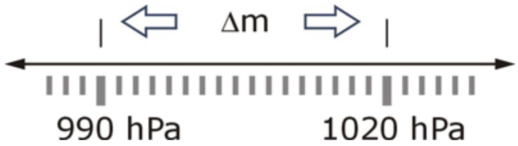


Fig. 4. Drift of mass in changeable atmospheric pressure

The size of possible error arising while weighing the same sample in specific time intervals depends on:

- changes of air density in time
- differences occurring between the density of a standard ($8000 kg/m^3$) and density of a sample.

Assuming that a standard change in air density is approximately 30 hPa, and the ambient temperature and humidity in a weighing room are constant, it is possible to assess the changes in mass of practically any sample. Such activities are worth carrying out, on condition, that sample's mass is monitored and recorded with an appropriate accuracy. Below tables present charts on air pressure drift in a short and long period of time.

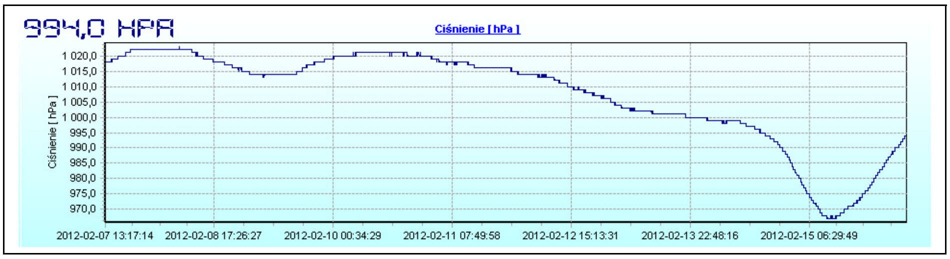


Fig. 5. Record from air pressure changes within period of time between 07/02÷ 16/02/2012

Total change of air pressure within the above period of time is 55 hPa, and in the period of time from 13th to 15th February the pressure has changed by 35 hPa.

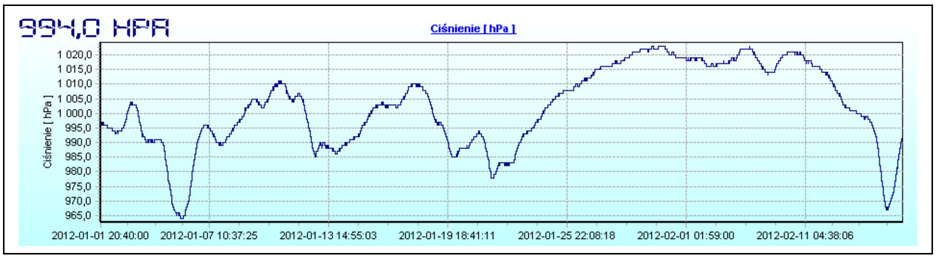


Fig. 6. Record from air pressure changes within period of time between 01/01÷ 16/02/2012

Record from air pressure changes in a longer period of time (47 days) shows, that the factor is subject of relatively high variability. It has to be considered while carrying out long term monitoring of sample's mass.

2.3. The value of buoyancy force during weighing materials with different density

The size of error contributed to the measurement result by the buoyancy force is assessable by assuming stability of all parameters in reference conditions. The difference between the mass indicated by a balance and corrected mass (by the buoyancy force) depends on the volume of weighed sample (its mass). If assumed that the air pressure is 1013 hPa, temperature = 20°C, humidity = 40%, below table presents the following values for different materials.

Sample mass	**Timber** 800 kg/m³	**Water** 1000 kg/m³	**Rubber** 1600 kg/m³	**Chalk** 2000 kg/m³	**Tantalum** 16600 kg/m³
	Deviation from the real (true) value [mg]				
0,001 g	0,001	0,001	0,000	0,000	0,000
0,01 g	0,013	0,010	0,006	0,003	0,000
0,1 g	0,135	0,105	0,060	0,033	- 0,008
1 g	1,352	1,051	0,600	0,330	- 0,078
10 g	13,520	10,513	6,005	3,302	- 0,777
20 g	27,041	21,025	12,009	6,603	- 1,554
40 g	54,081	42,051	24,018	13,206	- 3,109
60 g	81,122	63,076	36,027	19,810	- 4,663
80 g	108,163	84,101	48,036	26,413	- 6,217
100 g	135,203	105,127	60,045	33,016	- 7,772

Table 1. The influence of buoyancy force onto measurement of different samples

The tabular data is a source of relationship, which are described as:

- the lower sample density, the bigger deviation from the true value.
- The lower sample mass in a given density, the smaller deviation from the true value.

The true value is understood here as a measurement that is not affected by the buoyancy force, i.e. a measurement in vacuum. Below there is a graphic interpretation of the above approach, that is measurement in vacuum. Below there is a graphic interpretation of the approach.

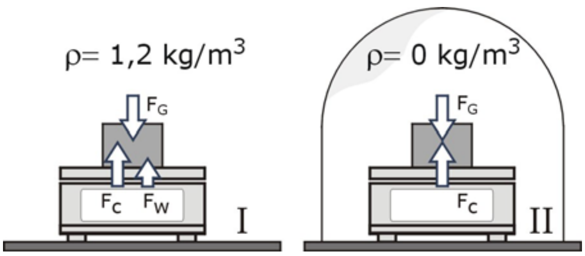
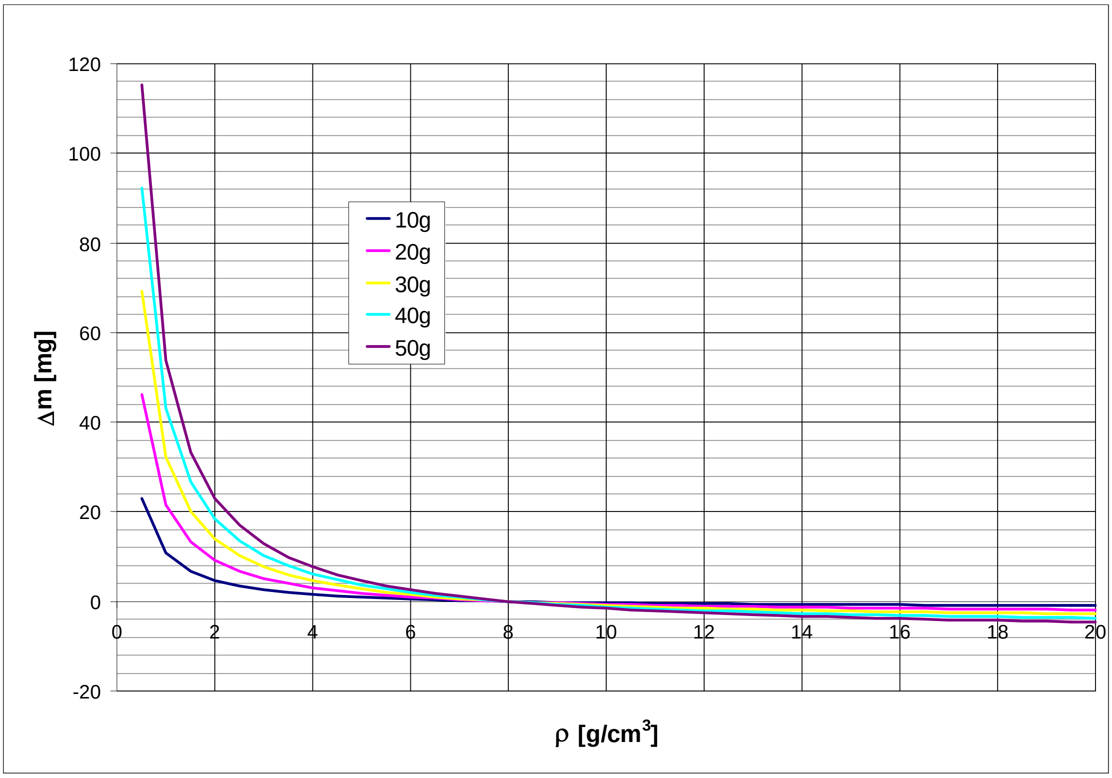


Fig. 7. I – measurement in ambient conditions, II – measurement in vacuum

Below there is a chart presenting relation between the size of correction (Δm) and different densities (ρ) and their mass.



Analysis of the chart provides a conclusion, that in case of materials having density equal to the density of a standard (8000 kg/m^3) used to adjust the balance, the value of correction Δm resulting from buoyancy force, is equal to zero.

2.4. Mass measurement of loads in variable ambient conditions

As demonstrated, the correction resulting from the presence of buoyancy force may be significant. The application of the correction is conditioned by the requirements on measuring accuracy in a weighing analysis. Therefore, each weighing process requires individual assessment of errors.

During measuring process, it is possible to determine two main areas which are source of errors. The first one is parameter of a balance, in particular repeatability of indications and stability of sensitivity. It is possible to use data supplied by balance's manufacturer or carry out adequate tests. As for the repeatability, it should be tested in actual operating conditions, with use of elements and samples that will be weighed.

The stability of sensitivity is maintained on an appropriately high level by balance's automatic adjustment system, which is a standard in RADWAG balances. A balance automatically carries out periodical adjustment which is triggered by temperature changes and time.

The other area refers to information on ambient conditions and the weighed sample, i.e.:

- error of temperature measurement
- error of humidity measurement
- error of atmospheric pressure measurement
- accuracy of determining sample’s density

The temperature, humidity and pressure determine air density, therefore by analyzing changes of these factors, one obtains uncertainty of air density determination. Accuracy of determining sample density is understood as reliable number of decimal places (e.g. information like 1300,1 kg/m^3 is insufficient).

2.5. INFLUENCE OF AIR BUOYANCY ON CALIBRATION OF MASS STANDARDS AND WEIGHTS

Calibration of mass standards and weights of high accuracy class requires accepting a correction arising from presence of aerostatic forces – so called air buoyancy correction. It stems from the fact, that during measurement process in the air, the measurement result is apparent, which in case of mass standards and weights with the same mass but made of different material (and different density) is not equal.

Mass measurement of two weights made of different materials gives non-equal measurement results. Thus, the procedure requires calculating the actual mass of the weights, i.e. calculating their mass in vacuum by introducing a correction resulting from air buoyancy *w* determined using the following formula:

$$W = w_B - w_k = (V_B - V_K) \cdot \rho$$

or

$$w = w_B - w_k = m_n \left(\frac{1}{\rho_B} - \frac{1}{\rho_k} \right) \cdot \rho$$

where:

w_B – mass of air displaced by the calibrated weight
w_k – mass of air displaced by the (reference) standard
V_B – volume of calibrated weight
V_k – volume of reference standard
ρ - density of air
ρ_B – density of calibrated weight
ρ_k – density og reference weight
M_n – nominal mass of calibrated weight

In practice, in case of mass standards and weights in accuracy class E_2, F_1, F_2, M_1, M_2 and M_3 the volume is not determined, but it is calculated on basis of the known material density used to manufacture the calibrated standards or weights. In majority of cases, the density of weights is unknown for specific nominals. It is so because usually the material for weights is not homogenous, the weights feature adjusting cavity and different finishing.

The correction arising from the difference in air buoyancy changes in relation to the density of air. Therefore, the ambient conditions should be maintained constant in a weighing room where the measurements are carried out.

It has been accepted, that "standard ambient conditions" are at air temperature 20 °C, relative air humidity 50 % and atmospheric pressure 1013,25 hPa. In case of such ambient conditions, the air density equals approximately 1,2 kg/m^3. Average deviation of air density maintains within the ± 10 % threshold, if change of air temperature does not exceed ± 15 °C and change of atmospheric pressure does not exceed ± 46 hPa. In case of such changes of air density, the correction sourcing from the difference in air buoyancy is $2\cdot10^{-6}$ mass of the compared weights. For instance, weights made of stainless steel, which density equals ~ 7900 kg/m^3 and brass, which density is ~ 8400 kg/m^3.

If uncertainty of determining mass of weights is approximately $1\cdot10^{-5}$, then the correction can be omitted. But, if the aim is obtaining the lower uncertainty, then the correction has to be considered. Due to the fact that the density of mass standards and weights is not precisely known, nor it is equal, the conventional density has been accepted, which is close to their nominal density.

Presently, the conventional density value accepted for the weights is approximately 8000 kg/m^3 while weighing in the air in ambient temperature 20 °C and average air density 1,2 kg/m^3.

In order to standardize mass measurement processes and maintain traceability in comparing indications and maximal permissible errors, the conventional density of weights of 8000 kg/m^3 and average air density of 1,2 kg/m^3 have been internationally adopted.

Adopting the conventional density of mass standards and weights has excluded the calculation of the correction resulting from difference in air buoyancy, and thus simplified calibration procedure. Weighs made of different materials (different volume – density), which mass has been determined using the conventional density 8000 kg/m^3, "compensate while weighing in the air".

In order to ensure sufficient measuring accuracy, it is required that the actual density of weights is maintained within strictly determined limits in relation to the conventional density. The density of mass standards and weights should be adjusted in such a way, that 10 % change of air density in relation to the average value of 1,2 kg/m^3 does not cause an error exceeding 0,25 of the maximal permissible error (Mpe) for the mass standard or for the weight (Table 1 of Document R111-1 OIML).

The OIML Recommendation R111-1 provides minimal and maximal limits for density of mass standards and weights:

Nominal value	ρ_{min}, ρ_{max} (10^3 kg/m^3)							
	Accuracy class of weights							
	E_1	E_2	F_1	F_2	M_1	M_{1-2}	M_2	M_{2-3}
≥ 100 g	7,934-8,067	7,81-8,21	7,39-8,73	6,4-10,7	≥ 4,4	≥ 3,0	≥ 2,3	≥ 1,5
50 g	7,92-8,08	7,74-8,28	7,27-8,89	6,0-12,0	≥ 4,0			
20 g	7,84-8,17	7,50-8,57	6,6-10,1	4,8-24,0	≥ 2,6			
10 g	7,74-8,28	7,27-8,89	6,0-12,0	≥ 4,0	≥ 2,0			
5 g	7,62-8,42	6,9-9,6	5,3-16,0	≥ 3,0				
2 g	7,27-8,89	6,0-12,0	≥ 4,0	≥ 2,0				
1 g	6,9-9,6	5,3-16,0	≥ 3,0					
500 mg	6,3-10,9	≥ 4,4	≥ 2,2					
200 mg	5,3-16,0	≥ 3,0						
100 mg	≥ 4,4							
50 mg	≥ 3,4							
20 mg	≥ 2,4							

Table 2. Minimal and maximal limits for density of mass standards

3. Testing and results of practical tests

3.1. Testing workstation

Testing the influence of changeable air density onto the weighing process requires a dedicated workstation which comprises of:

- hermetically sealed pressure chamber [1]
- electronic balance MYA 5/2Y [2]
- compressor [3]
- automatic system of placing tested mass [4]
- ambient conditions module: pressure, temperature, humidity, air density [5]
- system for remote controlling the balance’s operation and power supply [6]
- weighed object [7]
- sensor for controlling pressure [8]
- density determination KIT 85 for determining density of solids and liquids [9]

A scheme of a testing workstation:

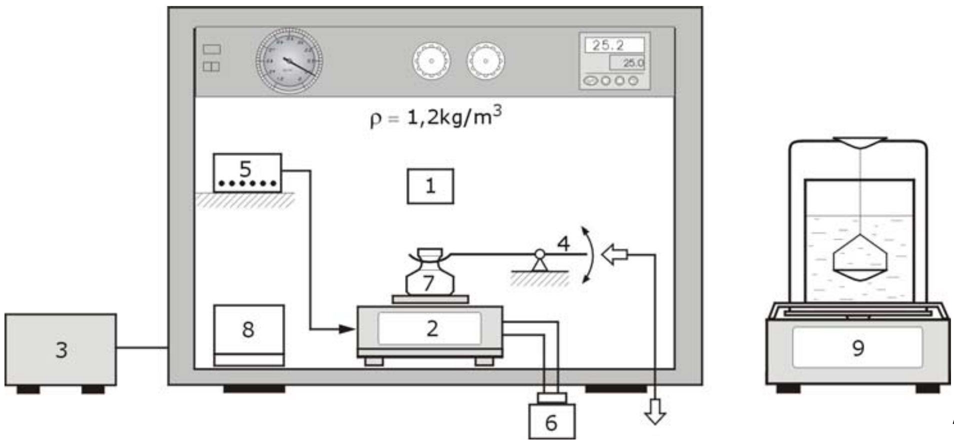


Fig. 8. A workstation for testing influence of buoyancy force on the weighing process

The test is to determine changes in mass of a sample in standard conditions (ρ=1,2kg/m^3) and in overpressure and negative pressure. The hermetically sealed chamber [1] is connected with a compressor [3], which maintains set pressure. A balance [2] acquires data on ambient conditions (pressure, humidity, temperature and air density) from connected ambient conditions module [5]. Based on such data it calculates coefficient correcting the influence of the buoyancy force onto the weighed sample [7]. The automatic system for placing tested mass [4] is controlled through an external signal, and it enables loading and unloading the sample [7] from balance’s weighing pan. The remote control system [6] records the measurement results and balance’s adjustment process. The density of samples is determined using the KIT 85 [9].

3.2. Test results

The purpose of the test was to prove the trueness of the theoretical assumptions describing the influence of buoyancy force on the measurement process, and evaluation of the functioning effectiveness of the corrective coefficient. Testing of the changeability of samples' mass in different air density in two systems:

- measurement without a coefficient correcting the buoyancy force
- measurement with a coefficient correcting the buoyancy force

The test is to demonstrate the contribution of buoyancy force in the process of weighing an aluminum sample with density 2700kg/m^3 and rubber sample with density 1367kg/m^3 if standard pressure changes occur. The tests were carried out by recording sample's mass in different atmospheric pressure. It has been assumed, that the maximal pressure change is 30 hPa. Before test initialization, the balance used for tests has been adjusted using its internal adjustment mass for determining its accuracy.

3.2.1. Mass change analysis of the aluminum sample

During the test no. 1 the corrective coefficient of the buoyancy force has not been used. Therefore the test gave two mass readings: m1 – sample mass while weighing in ambient pressure p1 and m2 – sample mass while weighing in pressure p1 – 30hPa. The results of test no. 1 are demonstrated in below table:

p1 = 996 hPa	p2 = 966 hPa	Δp = 30 hPa
m_{AL-1} = 3,591667 g	m_{AL-2} = 3,591688 g	Δm = 0,000021 g

The conclusion from the measurement data is that weighing the same sample in different pressure causes an error of indication 21 µg. This value may be telling while carrying out differential weighing of the sample, e.g. weighing analysis of wearing process of sample's surface, surface coating using protective or decorative layers.

The second phase of the test was to eliminate the change of mass resulting from changes in pressure. In this phase the corrective coefficient of buoyancy force was used. Correct application of the coefficient requires data on sample density and current air density. The density of a sample has been determined using KIT 85 which gave a result of 2,701167 g/cm3. Data on air density has been acquired online from the ambient conditions module. The results of test no. 2 are given in below table:

p1 = 996 hPa	p2 = 966 hPa	Δp = 30 hPa
m_{AL-3} = 3,592662 g	m_{AL-4} = 3,592664 g	Δm = 0,000002 g

The obtained difference between mass m1 and m2 demonstrates, that the corrective coefficient functions correctly. Thus, testing sample's mass in a long period of time shall not be burdened with an error caused by changing pressure. The deviation between mass m_{AL-3} and m_{AL-4} results from repeatability of balance's indications for tested mass and testing conditions. The difference between mass in test no. 1 and test no. 2 arises from application of the coefficient correcting influence of the buoyancy force.

3.2.2. Mass change analysis of the rubber sample

During the test no. 1 the corrective coefficient of the buoyancy force has not been used. Therefore the test gave two mass readings: m1 – sample mass while weighing in ambient pressure p1 and m2 – sample mass while weighing in pressure p1 – 30hPa. The results of test no. 1 are demonstrated in below table:

p1 = 996 hPa	p2 = 966 hPa	Δp = 30 hPa
m_{G-1} = 5,502322 g	m_{G-2} = 5,502425 g	Δm = 0,000103 g

The measurement of the same sample in different pressures was burdened with an error 103 μg. The difference in mass is larger than in case of weighing the aluminum sample, as:

- density of rubber is significantly lower than density of aluminum
- mass of rubber is higher than mass of aluminum.

The obtained difference may be related to the actual measured mass, bearing in mind that the dependence between error and mass is linear. Therefore, in case of a sample weighing 55g the obtained error is approximately 1 mg.

The second phase of the test no. 2, the coefficient correcting the influence of buoyancy force in measurements was applied. The obtained sample density was 1367 kg/m3. Data on air density has been acquired online from the ambient conditions module. The results of test no. 2 are given in below table:

p1 = 996 hPa	p2 = 966 hPa	Δp = 30 hPa
m_{G-3} = 5,506125 g	m_{G-4} = 5,506129 g	Δm = 0,000004 g

The difference between mass in both tests reached 4 μg and it is within the limits of balance's repeatability for applied load. The coefficient correcting the influence of buoyancy force functions correctly in different value of pressure.

The results obtained during test no.1 significantly differ from the results of test no.2. The difference arises from the fact, that during test no.1 the corrective coefficient has not been applied, and the obtained mass was a "conventional" one. During test no. 2 the coefficient has been applied, therefore each mass is a corrected one, as if the measurement was carried out in the vacuum.

3.2.3. Mass change analysis of filters used for measuring dustiness

Testing air dustiness with dust PM 2,5 or PM 10 requires using filters with relatively small diameter. Mass of such filters are within the range from approximately 50 mg to about 200 mg. The influence of buoyancy force on such small objects is usually neglected. However, such tests also use large filters, weighing over 3 grams. In such cases, the contribution of buoyancy force is significant, bearing in mind, that the result is a differential one from weighing tested mass.

For the purpose of testing the influence of buoyancy force, the operators used clean filters made of boron-silica glass type GF/A and GF/C weighing approximately 0,8 g. Thus, it has been assumed, that the influence of dust mass that is placed on a filter after taking the measurement using an aspirator is neglected (too low mass). The main error arises from the weighing process and occurs on changeable air density.

During test no. 1 the coefficient correcting buoyancy force was not applied. It enabled obtaining changeable mass of a filter in pressure changed by 30 hPa. The results of test no. 1 are demonstrated in below table:

p1 = 996 hPa	p2 = 966 hPa	Δp = 30 hPa
m_{F-1} = 0,875558 g	m_{F-2} = 0,875570 g	Δm = 0,000012 g

The measurement of filters' mass in different pressure value is burdened with an error reaching 12 μg. The obtained result, if referred to the filter's mass (approximately 3 g) gives a measurement error of approximately 40 μg. In any case of estimating whether this quantity is significant, it has to be referred to the quantity of absorbed dust.

During test no. 2 the coefficient correcting buoyancy force was applied. Determined sample density was 2,79 g/cm3, data on air density has been acquired from the ambient conditions module. The obtained measurement results are provided in below table:

p1 = 996 hPa	p2 = 966 hPa	Δp = 30 hPa
m_{F-3} = 0,875792 g	m_{F-4} = 0,875797 g	Δm = 0,000005 g

On applying the mechanism correcting filter's mass, the obtained difference in mass is 5 μg. In this case the improvement in stability is approximately 50 % and it strongly relates to the characteristics of a sample, in particular its size and internal structure.

4. Analysis of errors referring to determining corrected mass

Any process aiming at determining a quantity is burdened with errors sourcing from used measuring equipment, used probes and sensors, and selected measurement method. Estimating the true value of a quantity determined in time is possible on carrying out error analysis. It enables assessing usability of the measuring equipment and methods. Using above listed data additionally provides parameters for modifying a testing workstation, which aims at obtaining desired measurement accuracy.

4.1. Repeatability of balance indications, stabiluty of sensitivity

Repeatability of indications according to the International Vocabulary of Basic and General Terms in Metrology VIM 2010, is measurement precision under repeatability conditions of measurement, including:

- same procedure
- same operator
- same measuring system
- same operating conditions, the same location
- replicated measurements over a short period of time.

The practice of determining corrected mass, orders looking into the notion of repeatability through two aspects. In case of small mass, approximately 10% ÷ 20% Max it is the main component of measurement uncertainty, participation of other errors is neglected. Thus, it can be stated, that balance indication is true, but burdened with an error sourcing from repeatability of indications.

In case of large mass, apart from repeatability, one should also consider participation of other factors, like eccentricity or linearity. However, the accuracy of balance indications is mainly conditioned by two factors:

– the repeatability of indications, and
– accuracy class of mass standards (their errors), used for determining balance accuracy.

Generally, a statement is true: the better repeatability, the smaller random error δ. Graphic interpretation of the relation is presented in below graph:

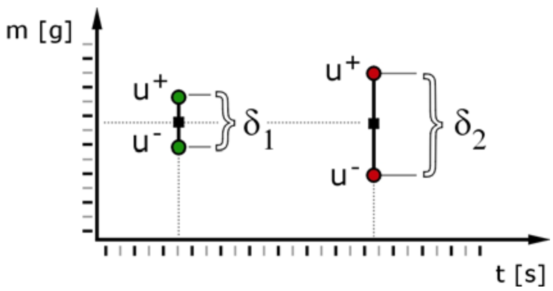


Fig. 9. Repeatability as a balance's parameter

The above drawings indicate, that the better repeatability (smaller error of a single measurement), the smaller the random error while weighing a sample.

$$\delta_1 < \delta_2$$

An improvement in repeatability can be achieved by optimizing balance parameters regarding the operating conditions.

Stability of sensitivity [ΔS] is, in other words, balance ability to indicate the same and constant measurement result for the same mass, which is weighed in specific time intervals. If assumed, that stability of sensitivity is ideal ($\Delta S = 0$) the only source of errors would be e.g. repeatability of indications (measurement uncertainty).

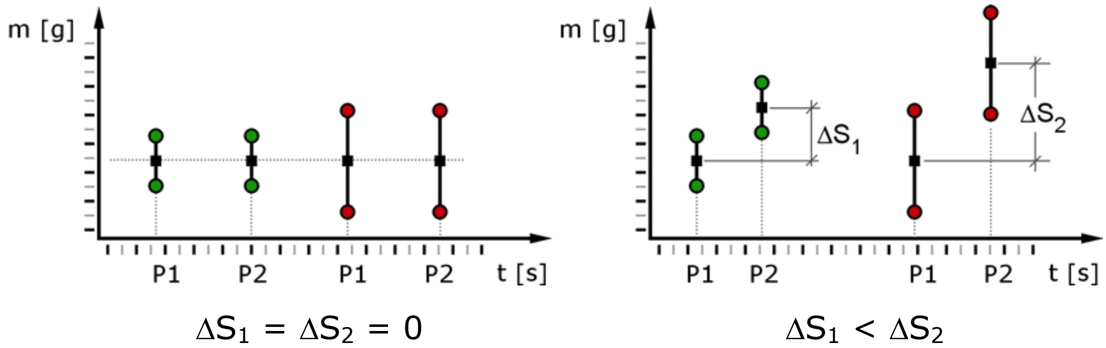


Fig. 10. Sensitivity stability in time

However, in practice there are no ideal instruments, therefore a balance having a better stability of sensitivity (ΔS_1) contributes to the final result of differential analysis much smaller error. The error arising from change in sensitivity (drifts) is eliminated by balance's adjustment system. Good Practice orders carrying out instrument's adjustment before starting the testing measurements.

4.2. Influence of temperature measurement error on determining air density

The temperature measuring system used in a balance enables its controlling with accuracy of ±0,2°C. Therefore, it has to be assessed how such temperature dispersion influences the process of determining air density, and thus coefficient correcting buoyancy force.

Use of a formula [1] allows determining how a temperature error of 0,4 °C in relation to the true value (other factors constant) influences air density determining process. The obtained difference in air density is 0,0017725 kg/m^3.

Having the extreme values of air density, it is possible to determine error of mass measurement using formula [2]. Measurement of a model material with density 2500kg/m^3 provides the following differences in measurements:

Sample mass [g]	δ m [μg]
0,001 g	0
0,01 g	0
0,1 g	0
1 g	0,5
10 g	4,9
20 g	9,8
40 g	19,50
60 g	29,30

Table 3. Influence of temperature measurement inaccuracy on mass measurement error

As demonstrated in the table, the measurement error resulting from inaccuracy of temperature measurement only is rather small. In case of samples weighing 60 g it is maximally 0,03 mg. This quantity remains most likely undetected, as repeatability determined as standard deviation for balances with reading unit d= 0,01mg is within 0,015 ÷ 0,03 mg.

The uncertainty of temperature measurement definitely depends on the initial assumptions, i.e. defining the required accuracy of measurement process.

4.3. Influence of humidity measurement error on determining air density

The humidity measuring system used in a balance enables its controlling with accuracy of 0,05%, however, the accuracy parameter has a tolerance of ± 1,8%. It results from sensor's hysteresis, thus during analysis it has been assumed that the measurements can be burdened with an 3,6% error. Using formula no. [1] and no. [2] it is possible to determine influence of humidity onto measurement of air density, and error of mass measurement. For a model material with density 2500kg/m^3 depending on mass provides the following differences in measurements:

Sample mass [g]	δ m [μg]
0,001 g	0
0,01 g	0
0,1 g	0
1 g	0,1
10 g	1
20 g	2
40 g	4,1
60 g	6,1

Table 4. Influence of humidity measurement inaccuracy on mass measurement error

Humidity has relatively small influence on air density if compared to temperature influence.

Analysis of obtained results is used to form the following statements:

- influence of temperature measurement error on determining air density is relatively small. For a model mass 10g, which is coceived as large in case of microbalances, the obtained error is approximately 5μg (standard deviation for such mass is approximately 2 μg). in case of 100 g mass (density 2500 kg/ m^3) the error of indication is 0,04 mg.

- influence of humidity measurement error on determining air density can be considered as negligible. Errors sourcing from this factor are much smaller than repeatability of balance's indications.

4.4. Influence of pressure measurement error on determining air density

The measurement of air density in RADWAG balances is carried out using sensors with measuring accuracy ±0,2 hPa. Therefore, it should be assumed, that during a measurement possible maximal deviation from the true value is 0,4 hPa. This change may cause error of determining air density by 0,00047 kg/m^3. In case of a model material with density $2500 kg/m^3$ and depending on weighed mass, the following differences in measurements are achieved:

Sample mass [g]	δ m [μg]
0,001 g	0
0,01 g	0
0,1 g	0
1 g	0,1
10 g	1,3
20 g	2,7
40 g	5,2
60 g	7,9

Table 5. Influence of pressure measurement inaccuracy on mass measurement error

The applied pressure measuring sensor features sufficient accuracy. The error sourcing from incorrect pressure indication is considerably smaller than the error sourcing from balance repeatability of indications for given mass – similarly as in case of temperature and humidity measurement. Thus, detection of an error generated by the pressure measuring sensor in mass measurement in practically impossible.

4.5. Influence of ambient conditions measurement accuracy-summary

The applied temperature, humidity and pressure measuring systems have small influence on determination of air density, and thus for corrected mass measurement. The size of errors contributed to the weighing process are much smaller than balance's repeatability for a given load. Therefore, detection of the changes through weighing seems to be problematic. Below table presents summary for a few weighed loads.

Sample mass [g]	Mass measurement error δm arising from measurement inaccuracy			Balance parameters d/sd
	Temperature [±0,2°C]	Humidity [± 1,8%]	Pressure [±0,2 hPa]	
100 mg	0,0 μg	0,0 μg	0,0 μg	0,1 μg/ 1 μg
1 g	0,05 μg	0,1 μg	0,1 μg	0,1 μg / 1 μg
20 g	9,8 μg	2 μg	2,7 μg	1 μg / 3 μg
60 g	29,3 μg	6 μg	7,9 μg	10 μg / 30 μg

Table 6. Influence of ambient conditions measurement inaccuracy on mass measurement error

The table additionally lists the value of balance's reading unit [d] and repeatability expressed as standard deviation [sd]. It is assumed, that the balances used for measurements have maximal capacity close to weighed loads.

Standard deviation is a measure of balance's repeatability of indications, and it is determined using a formula:

$$s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n-1}}$$

where:

s – standard deviation

x_i – following measurement

$\overline{x}$ – arithmetic mean from series of measurements

n – number of repetitions in a series of measurements

The actual dispersion of balance indications is three times higher than the standard deviation. It is a form of approximation, and the exact value should be determined through tests.

4.6. Influence of accuracy of sample density on indication corrected by buoyancy force.

One piece of information required during calculating mass corrected by buoyancy force is sample's density. While assessing how accuracy of sample's density influences on value of corrected mass, it is possible to to use below formula [2]:

$$m_c = m \cdot \frac{1 - \frac{\rho_0}{\rho}}{1 - \frac{\rho_0}{\rho_c}}$$

where: m_c – sample mass
ρ_0 – air density [kg/m^3]
ρ – density of a standard used to adjust a balance [8000 kg/m^3]
ρ_c – density of weighed sample [kg/m^3]
m – measurement result indicated by a balance

On assuming, that actual (true) density of a sample is 1600 kg/m^3, and the error of its determination is 0,01 g/cm^3 and 0,1 g/cm^3 (all other factors are constant) the obtained dispersion of determining corrected mass is as in below table.

Sample mass	Deviation from true value if density error amounts for the following:	
	0,01 g/cm^3	0,1 g/cm^3
0,001 g	0,0 mg	0,0 mg
0,01 g	0,0 mg	0,0 mg
0,1 g	0,001 mg	0,004 mg
1 g	0,005 mg	0,044 mg
10 g	0,046 mg	0,441 mg
20 g	0,093 mg	0,882 mg
40 g	0,186 mg	1,764 mg
60 g	0,280 mg	2,647 mg
80 g	0,373 mg	3,529 mg
100 g	0,466 mg	4,411 mg

Table 7. Influence of sample density inaccuracy on mass measurement error

Bearing in mind, that the error sourcing from inaccuracy of ambient conditions measurement is contained within the limits from a few to dozens of micrograms (see table 6), the critical importance while determining corrected mass is on correct specification of sample's density.

5. Means of correcting buoyancy force used in RADWAG balances

The function correcting the buoyancy force requires current data on the density of air and the density of weighed sample. Based on these two parameters it is possible to determine the corrective coefficient. Thus, during measurement the only contribution relates to the force of gravity, in accordance to a formula:

$$F = m \cdot g$$

RADWAG balances 2Y series use two methods of correcting the buoyancy force. Below points presents both methods, their means of operation and relation between each of them.

5.1. Method 1 – semi automatic with application of mass standards

It is a dual stage method requiring determining air density through weighing two mass standards (made of steel and aluminum) of the same mass, but different density. Then, the calculated value should be applied in a function of buoyancy force correction. Below graph presents sequence of activities in this method.

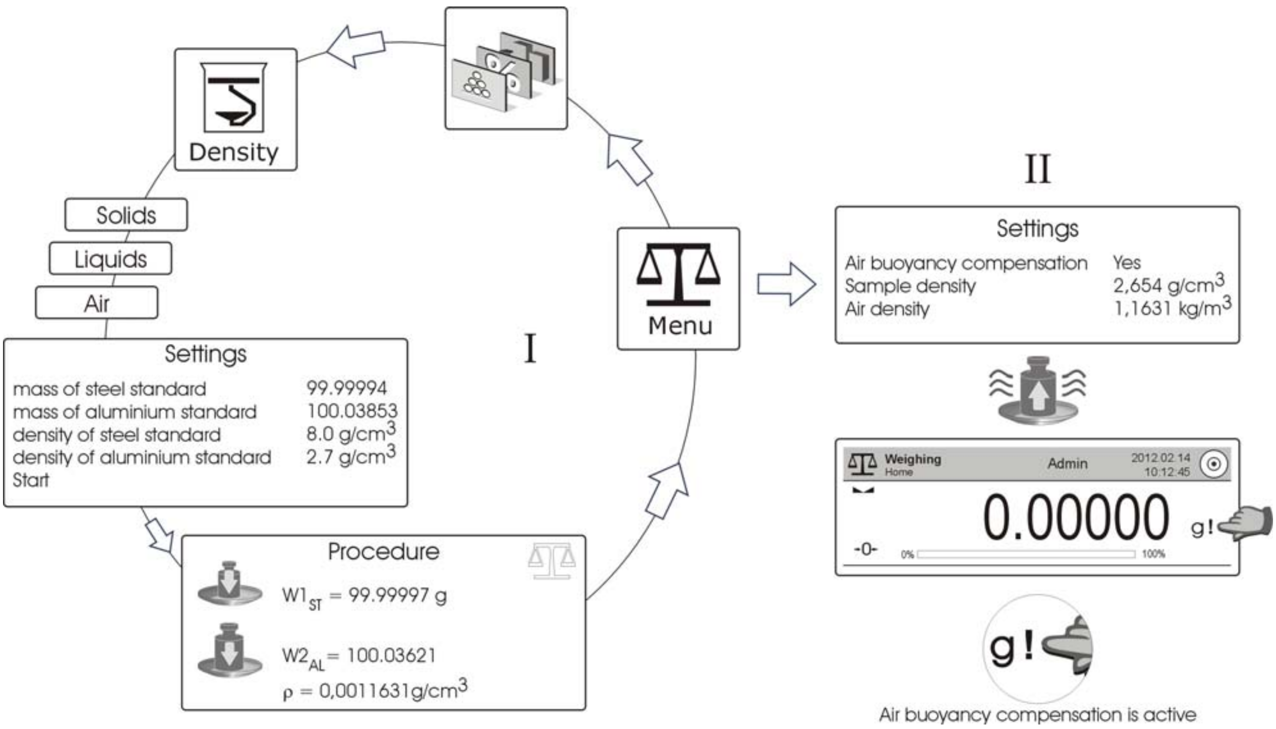


Fig. 11. Sequencing in Method no. 1

This method comprises three basic components, which may be sources of possible uncertainty. The first one is data on density of mass standards and their mass. Density of mass standards should be determined by testing, thus making the results reliable.

Such approach is obligatory, as it is used for calculating the density of air. Both density and mass of standards are specified on the calibration certificate. For obvious reasons, the calibration should be carried out by an Accredited Laboratory. In case of specific analysis, it is possible to evaluate the influence of uncertainty of determining the values on the final result of the measurement.

Set of mass standards for determining density of air
(1) – aluminum standard, (2) – stainless steel standard

The second area is measuring mass of the two standards. Assuming, that sensitivity of a balance is correct (automatic adjustment is enabled), the only important parameter is balance's repeatability of indications. In RADWAG balances this parameter does not exceed 1 reading unit. Air density is calculated using a relation:

$$\rho_a = \frac{m_{AL} \cdot W_{ST} - m_{ST} \cdot W_{AL}}{\frac{m_{AL} \cdot W_{ST}}{\rho_{AL}} - \frac{m_{ST} \cdot W_{AL}}{\rho_{ST}}}$$

where:

m_{AL} – corrected mass of aluminum standard
m_{ST} – corrected mass of stainless steel standard
W_{ST} – weighing result of steel standard
W_{AL} – weighing result of aluminum standard
ρ_{AL} – density of aluminum standard (2.7 g/cm^3)
ρ_{ST} – density of steel standard (8.0 g/cm^3)

The corrected mass of aluminum and steel standard is determined using below formulas:

$$m_{ST} = M_{ST} \cdot \frac{1 - \frac{1{,}2 kg/m^3}{8000 kg/m^3}}{1 - \frac{1{,}2 kg/m^3}{\rho_{ST}}} \qquad m_{AL} = M_{AL} \cdot \frac{1 - \frac{1{,}2 kg/m^3}{8000 kg/m^3}}{1 - \frac{1{,}2 kg/m^3}{\rho_{AL}}}$$

where:
M_{ST} – weighing result of steel reference mass
M_{AL} – weighing result of aluminum reference mass

It can be assumed, that in perfect conditions, weighing result of reference mass is equal to the value of reference mass.

The third area is use of determined air density for the function of buoyancy correction. In principle it is only calculation of values, and a potential error can be excluded here. Analysis of density determining method draws a conclusion, that its most important component is measurement process, which determines the accuracy of the method.

5.2. Method 2 – automatic with application of sensors

Technical solution used in this method is a new and innovative one in designing laboratory balances. RADWAG as the first manufacturer of laboratory balances applied electronic system for measuring pressure using sensors integrated with the balance. In this method, the only parameter requiring input is sample's density. All other parameters referring to ambient conditions are. Data on temperature, humidity, pressure and air density can be:

- sent from sensors installed inside balance's housing, or
- acquired from an external THB ambient conditions module. The THB module is a mobile measuring system, connected to balance's interface.

A sequence of activities in Method 2.

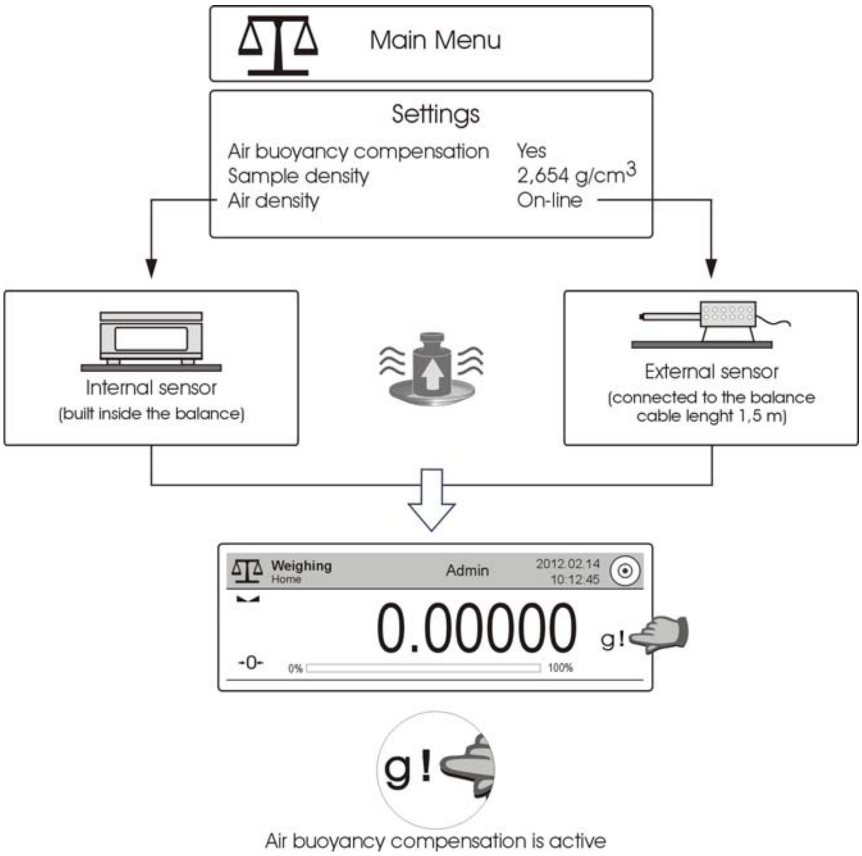


Fig. 12. Sequencing in Method no. 2

The balance's ambient conditions module can operate both internal or external sensors.

In such case, the external module can be also used to measure temperature and humidity in other section of a Laboratory.

Length of a cable connecting the external THB ambient conditions module with a balance is 1,5 m. An additional highlight of the external THB module is its connectivity to a computer software. Below figure demonstrates balance's display with enabled internal ambient conditions sensors and connected external ambient conditions module.

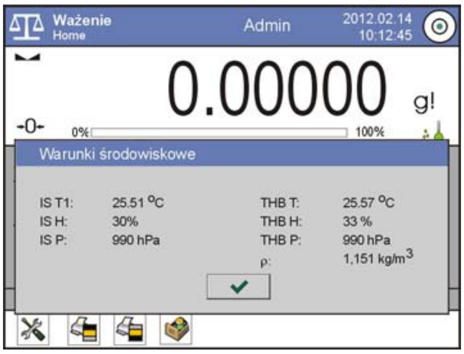


Fig. 13. Balance display with enabled ambient conditions panel

The internal THB sensors are marked as IS, and the external sensors are denoted as THB. Additionally, balance's software provides data on current air density marked with ρ symbol. The density of air is calculated using data from the external ambient conditions module. In case the external THB ambient conditions module is disabled (not connected), the same information is calculated using parameters from internal ambient conditions sensors.

No connection, missing cables or additional components makes it advisable to use the internal sensors only. One limitation is that the measurement takes place inside balance's housing. Bearing in mind the mobility of the THB ambient conditions module, its application in multiple balances 2Y series, and comprehensive measurement of ambient parameters, it seems to be far better solution. As indicated, the choice of solution depends on measuring requirements of an organization.

Independently on pros and cons of both solutions, balances MYA /2Y and XA /2Y series come standard with internally installed ambient conditions sensors. It does not, however, exclude parallel application of the external sensors. In case of balances AS /2Y the only option is use of external THB ambient conditions module, as this series does not feature internally installed sensors.

Balance user can determine coefficient correcting the influence of buoyancy force on enabling the function as follows:

1. give density of weighed sample, and balance software automatically calculates the value of air buoyancy correction using determined air density.

2. give density of weighed sample and density of air. Balance's software automatically calculates air buoyancy correction value. In this case the data on density of air can be acquired from other sources (instruments, parameters from meteorological station, etc.).

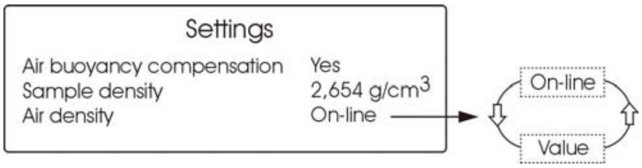


Fig. 14. Source of data for readout of ambient conditions

5.3. Calibration of sensors

In standard version, the sensors of temperature, humidity and pressure are not calibration. During manufacturing process, their correctness of indications is referred to calibrated instruments measuring temperature-humidity-(barometric) pressure (THB). Where necessary, it is possible to calibrate:

- internal sensors (built in balance's housing) in a balance. Calibration refers to the complete instrument, thus a complete balance should be delivered to a Calibration Laboratory as a one set,

- external THB ambient conditions module. In this case the calibration refers to the sensors only, as balance's display serves as information panel only. Thus, it is possible to use a calibrated ambient conditions module, applicable to various types of balances.

5.4. Balances with integrated ambient conditions sensors

Internal integrated sensors of temperature, humidity and pressure are installed in balances 2Y series with reading unit 0,1 mg and lower. Reading from an external pressure sensors requires connecting a THB-2 ambient conditions module. It is an optional equipment of laboratory balances.

Microbalance MYA / 2Y series

Technical data

Max 2 g ÷ 21 g
d = 0,1 μg ÷ 1 μg
e = 1 mg
Interface:
2×USB, 2×RS 232,
Ethernet,
2input/2output

Microbalances comprise two major components (an electronic module and a precise mechanical measuring system are enclosed separately). Such design eliminates the influence of heat sourcing from instrument's electronics on its mechanical components. Microbalances, apart from multiple applications, feature fully automatic internal adjustment system. The weighing chamber is made of antistatic glass. Two IR sensors are user programmable, as one of parameters enabling full personalization of balance settings.

Analytical balance XA / 2Y series

Technical data

Max 52 g ÷ 310 g
d = 0,01 mg ÷ 0,1 mg
e = 1 mg ÷ 10 mg
Interfejs:
2×USB, RS 232,
Ethernet, 2 inputs and
2 outputs

Analytical balances feature large weighing chamber with automatically opened doors. The XA/2Y series is controlled through a weighing module comprising a 5,7” colour display with touch panel. The balance’s terminal can be located next to balance’s housing, as it features a 0,3 meter long cable. Automatic internal adjustment system is a standard in this series of balances.

Technical data

Max 110 g ÷ 510 g
d = 0,1 mg
e = 10 mg
Interfejs:
2×USB, RS 232, Ethernet, 2 inputs and 2 outputs

Analytical balance AS / 2Y series

Ambient conditions in a balance AS /2Y series can be supervised only through application of an external THB ambient conditions module. It is the main difference occurring between this type of balance and MYA or XA series. The THB module is connected to a balance through RS 232 interface, and parameters on ambient conditions are acquired automatically with simultaneous visualization. Balances AS/2Y series feature fully automatic internal adjustment system, the head with display is installed on a flexible cable (for placing the head next to the balance's housing).

Technical data

- temperature measurement accuracy 0,2°C, probe length 95 mm;
- humidity measurement accuracy 1,8%;
- pressure measurement accuracy 0,2 hPa
- cable length 1,5 m
- slot RS 232 (DB 9)

External ambient conditions module THB

5.5. Comparison of methods

The major difference between the two methods is the means of determining the density of air. Method no. 1 uses two mass standards that have to be weighed. In this case, some influence on final result of air density is on repeatability of balance indications – there are two measuring process of mass standards.
Applying user's data on measurements or manufacturer's technical data enables assessing size of error which may occur as a result of carried out measurements. It is assumed that during measurements, the mass standards are placed in the center of balance's weighing pan, and the balance's sensitivity is properly adjusted.

Method no. 2 uses pressure sensors installed in balance's housing or external modules. Therefore, pressure readout is a result of an electronic measurement, which is then processed. As a result, the obtained value is air density. The accuracy of air density measurement is conditioned by class of installed pressure sensor. For obvious reasons, method no. 2 is faster and less complicated. Below table demonstrates comparison of both methods in details.

Method no. 1	**Method no. 2**
Uses mass standards to determine air density	Uses internal or external sensors to determine air density
Two-step procedure: weighing mass standards and enabling buoyancy compensation function	Single-step procedure: enabling buoyancy compensation function
Multiple determination of air density	Online operation (control of density takes place in 1 minute interval)
The need to have mass standards (steel and aluminum standards)	N/A
Influence of repeatability of balance's indications on measurement results during weighing process	N/A
Speed (the need to use intermediate functions – air density)	Speed (immediate with application of dedicated function)

Comparison of both methods gives clear conclusion, that method no. 2 is a highly recommended solution. It features innovative solutions which enable shortening and simplifying all activities related to precise determination of mass. It is less susceptible to operator's error associated with the weighing process. It eliminates potential errors of mass standards, which may appear in method no. 1.

6. Summary

The mechanisms enabling indicating measurement results corrected by buoyancy force are still not in common use, even though laboratory balances offer such option for fairly long time. As yet, the problem was how to implement the correction by using mass standards. Two staged procedure is not conceived as encouraging for majority of users. The economical aspect was also worth mentioning, i.e. the additional cost of purchasing and maintaining mass standards. Another and independent issue was insufficient number of publications describing the process of determining mass corrected by buoyancy force.

RADWAG by introducing new technical solutions in its laboratory balances formed a user friendly and ergonomic weighing applications presenting their operation in the context of existing physical phenomena. Such activities RADWAG transfers its knowledge, which if correctly used successfully solves problems occurring during mass measuring processes.

References:

[1] EURAMET/cg-18v.02 Guidelines on the Calibration of Non-Automatic Weighing Instruments
[2] OIML D-28 "Conventional value of the result of weighing in air"
[3] Piotrowska B., Odważniki Przepisy i komentarze, („Weights, Formulas and Comments") Wydawnictwo ALFA, Warszawa 1986
[4] OIML R 111-1 Edition 2004 Weights of classes E_1, E_2, F_1, F_2, M_1, M_{1-2}, M_2, M_{2-3} and M3, Part 1: Metrological and technical requirements