Validation of shape corrections in magnetic susceptibility and polarization measurements of weights

Tadeusz Szumiata[1], Michał Solecki[2], and Cezary Tomaszewski[3]
Centrum Metrologii Badań i Certyfikacji, Radwag Wagi Elektroniczne, Polska
e-mail: radom@radwag.pl; http://radwag.com, tel. +48 48 386 60 00

[1] University of Technology and Humanities in Radom, Faculty of Mechanical Engineering, Department of Physics, Stasieckiego 54, 26-600 Radom, Poland, t.szumiata@uthrad.pl
[2] RADWAG Balances & Scales, Toruńska 5, 26-600 Radom, Poland, m.solecki@radwag.pl
[3] RADWAG Balances & Scales, Toruńska 5, 26-600 Radom, Poland, tomaszewski@radwag.pl

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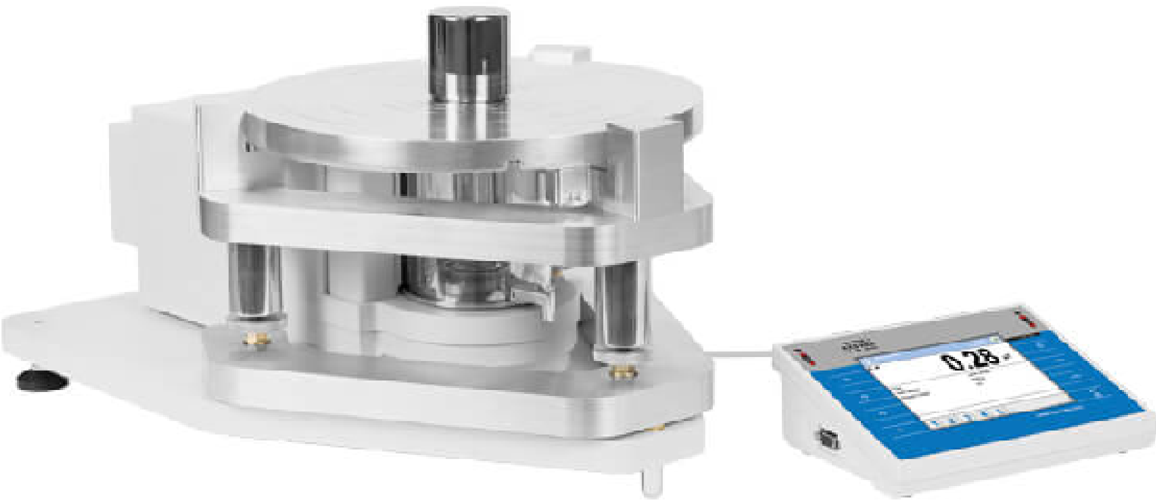

The OILM standard (Annex B, point B.6.4), recommends susceptometer method for determination of magnetic susceptibility and the permanent magnetic polarization of weights and mass standards.

The susceptometer (schematically presented in Fig. 1) consists of:

1. a weighing instrument with a scale interval not larger than 10 µg,
2. a non-magnetic table to place the weight on,
3. a non-magnetic cylinder (pedestal) to place the magnet on,
4. a small but strong cylindrical magnet (Sm-Co, Nd-Fe-B).

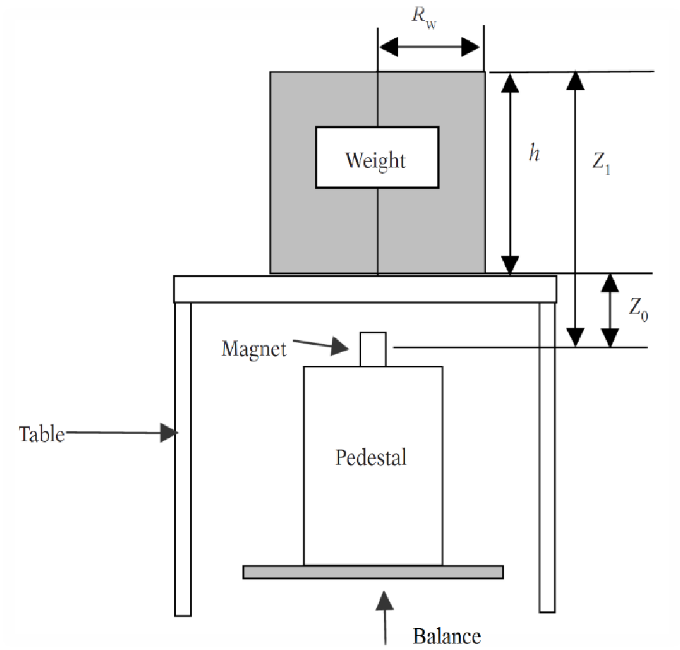

Fig. 1. Magnetic susceptometer method, scheme from OIML R 111-1 standard.

The measuring procedure demands comparison of balance indications for two opposite orientations of magnet (north pole down and up, respectively).There are several different practical realizations of suceptometers dedicated for checking the magnetism of weights. One of them is a susceptometer recently developed by RADWAG (Fig. 2). The distinguishable feature of this apparatus is implementation of software for magnetic susceptibility and polarization calculations directly in the microbalance indicator. Thus, RADWAG susceptometer is able to operate as an independent device without any need of utilizing the external computer. However, use of a dedicated PC software is also available.

Fig. 2. Magnetic susceptometer designed by RADWAG

Davis theory (OILM standard, Annex B, point B.6.4) predicts the following formulae for calculation of the magnetic susceptibility χ and the permanent magnetic polarization $\mu_0 M_Z$ of weight in the case of <u>cylindrical shape</u>:

$$\chi = \frac{F_a}{I_a \cdot F_{max} - 0.4 \cdot F_a} \tag{1}$$

$$\mu_0 M_Z = \frac{F_b}{\frac{m_d}{Z_0} \cdot \frac{1}{4\pi} \cdot I_b} - \frac{\chi}{1 + 0.23 \cdot \chi} \cdot B_{EZ} \tag{2}$$

The quantities F_a and F_b are defined as follows:

$$F_a = \frac{F_1 + F_2}{2} \quad \text{and} \quad F_b = \frac{F_1 - F_2}{2} \,, \tag{3}$$

where F_1 and F_2 are forces acting on magnet for two opposite orientations of magnetic moment ($+m_d$ and $-m_d$, respectively) of the permanent magnet. Z_0 is the distance from the mid-height of magnet to the bottom of the weight. The quantity F_{max} is described by the following formula:

$$F_{max} = \frac{3 \cdot \mu_0}{64 \cdot \pi} \cdot \frac{m_d^2}{Z_0^4} \tag{4}$$

where μ_0 denotes magnetic permeability of the vacuum.

The quantities I_a, I_b are geometrical (size) correction coefficients for the cylindrically shaped weights:

$$I_\mathrm{a} = 1 - \left(\frac{Z_0}{Z_1}\right)^4 - \frac{1 + \frac{\left(\frac{R_\mathrm{w}}{Z_0}\right)^2}{3}}{\left[1 + \left(\frac{R_\mathrm{w}}{Z_0}\right)^2\right]^3} + \left(\frac{Z_0}{Z_1}\right)^4 \cdot \frac{1 + \frac{\left(\frac{R_\mathrm{w}}{Z_1}\right)^2}{3}}{\left[1 + \left(\frac{R_\mathrm{w}}{Z_1}\right)^2\right]^3} \tag{5}$$

$$I_\mathrm{b} = 2\pi \cdot \left\{ \frac{\left(\frac{R_\mathrm{w}}{Z_0}\right)^2}{\left[1 + \left(\frac{R_\mathrm{w}}{Z_0}\right)^2\right]^{\frac{3}{2}}} - \frac{\frac{\left(\frac{R_\mathrm{w}}{Z_0}\right)^2}{\left(\frac{Z_1}{Z_0}\right)^3}}{\left[1 + \frac{\left(\frac{R_\mathrm{w}}{Z_0}\right)^2}{\left(\frac{Z_1}{Z_0}\right)^2}\right]^{\frac{3}{2}}} \right\} \tag{6}$$

where $Z_1 = Z_0 + h$ is the distance from the top of weight to mid-height of the magnet and R_w is the radius of the cylindrical weight.

The typical mass standards are cylinders, whereas calibration weights take forms strictly defined in the OIML R 111-1 standard (Fig. 3a).

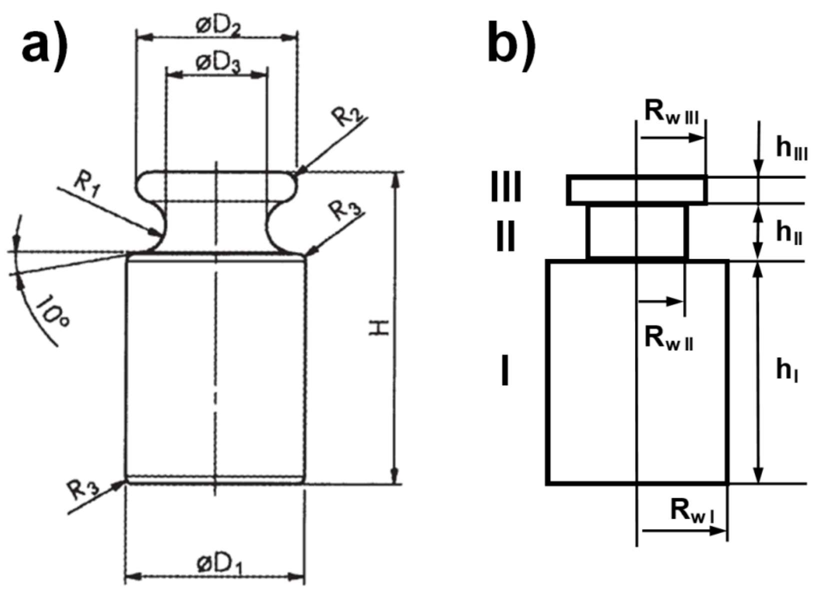

Fig. 3. a) Scheme of the axisymmetric weight, taken from OIML R 111-1 standard,
b) Scheme of 3-cylinder effective weight.

The realistic shape of weight can be simplified utilizing 3-cylinder effective model (Fig. 3b), which is more convenient for further analysis of weight magnetism.

Typical dimensions of real 50 g weight:

- $D_1 = 18.0$ mm,
- $D_2 = 16.0$ mm,
- $D_3 = 10.0$ mm,
- $R_2 = 1.5$ mm,
- $R_3 = 1.0$ mm,
- $H = 26.3$ mm.

The dimensions of simplified 3-cylinder equivalent:
$h_{\mathrm{II}} = 2 \cdot R_3 = 2.0$ mm, $h_{\mathrm{III}} = 2 \cdot R_2 = 3.0$ mm, $h_{\mathrm{I}} = H - (h_{\mathrm{II}} + h_{\mathrm{III}}) = 21.3$ mm. Moreover $R_{\mathrm{wI}} = D_1/2 = 9.0$ mm, and:

$$R_{\mathrm{wII}} = \frac{D_3}{2} + \frac{1}{6\left(1 - \frac{1}{4 \cdot \pi}\right)} R_3 = 5.2 \text{ mm}, \quad R_{w\mathrm{III}} = \frac{D_2}{2} + \frac{4}{3\pi} R_2 = 8.6 \text{ mm} \tag{7}$$

(considering the center of mass of the round profiles of radii R_3 and R_2 - Fig. 3a.).

In the case of 1-cylinder effective model the volume of one effective 1-cylinder should be the same as of 3-cylinder system, thus:

$$\pi R_{\mathrm{w}}^2 \cdot h = \pi R_{\mathrm{wI}}^2 h_{\mathrm{I}} + \pi R_{\mathrm{wII}}^2 h_{\mathrm{II}} + \pi R_{\mathrm{wIII}}^2 h_{\mathrm{III}} \tag{8}$$

$$h = h_{\mathrm{I}} + \frac{R_{\mathrm{wII}}^2}{R_{\mathrm{w}}^2} h_{\mathrm{II}} + \frac{R_{\mathrm{wIII}}^2}{R_{\mathrm{w}}^2} h_{\mathrm{III}} = 24.7 \text{ mm} \tag{9}$$

If one intends to go beyond 1-cylinder magnetic approximation, it is necessary to extend the model described by formulae (1-4). Within <u>3-cylinder magnetic model</u> the total force acting on the permanent magnet is the sum of forces coming from the magnetic field generated by each three magnetized, cylindrical parts of weight (Fig. 3b). Thus, the generalized formula for magnetic susceptibility can be written as:

$$\chi_{\mathrm{corr}} = \frac{F_{\mathrm{a}}}{I_{\mathrm{aI}} \cdot F_{\mathrm{maxI}} + I_{\mathrm{aII}} \cdot F_{\mathrm{maxII}} + I_{\mathrm{aIII}} \cdot F_{\mathrm{maxIII}} - 0.4 \cdot F_{\mathrm{a}}} \tag{10}$$

where I_{aI}, I_{aII}, I_{aIII}, F_{maxI}, F_{maxII}, F_{maxIII} are the analogous quantities to those previously defined, but at present they refer to I, II and III cylinders (describing real weight), respectively. By analogy, the extended formula for the permanent magnetic polarization takes the following form:

$$\mu_0 M_{\mathrm{Zcorr}} = \frac{F_{\mathrm{b}}}{\frac{m_{\mathrm{d}}}{Z_{0\mathrm{I}}}\cdot\frac{1}{4\pi}\cdot I_{\mathrm{bI}} + \frac{m_{\mathrm{d}}}{Z_{0\mathrm{II}}}\cdot\frac{1}{4\pi}\cdot I_{\mathrm{bII}} + \frac{m_{\mathrm{d}}}{Z_{0\mathrm{III}}}\cdot\frac{1}{4\pi}\cdot I_{\mathrm{bIII}}} - \frac{\chi_{\mathrm{corr}}}{1+0.23\cdot\chi_{\mathrm{corr}}}\cdot B_{\mathrm{EZ}} \tag{11}$$

where I_{bI}, I_{bII}, I_{bIII} are defined for I, II and III cylinders – similarly to I_{b} coefficient, which was previously introduced for one cylinder. In the same manner the quantities $Z_{0\mathrm{I}}$, $Z_{0\mathrm{II}}$ and $Z_{0\mathrm{III}}$ refer to each of the 3 cylinders. The presented approach is valid only, when the magnetic susceptibility is homogeneous over the whole weight, i.e. its value is the same in all cylindrical parts.

One of the most important goals of our considerations is to compare the results for magnetic susceptibility and permanent magnetic polarization determined with 1-cylinder model (formulae 1-2) and with 3-cylinder model (formulae 5-6). Let's assume, that in the case of the said 50 g calibration weight, the indications read from microbalance for two opposite orientations of magnet in susceptometer are following:

$$\Delta m_1 = -0.5430 \text{ mg} \quad \text{and} \quad \Delta m_2 = -0.1247 \text{ mg}.$$

Thus, corresponding forces acting on the magnet are:

$$F_1 = -\Delta m_1 \cdot g = 5.327\cdot 10^{-6} \text{ N and } F_2 = -\Delta m_2 \cdot g = 1.223\cdot 10^{-6} \text{ N},$$

where $g = 9.81$ m/s^2 is a value of local acceleration of gravity. Thus, according to definitions in section 3, one obtains: $F_{\mathrm{a}} = 3.275\cdot 10^{-6}$ N and $F_{\mathrm{b}} = 2.052\cdot 10^{-6}$ N.

A typical magnetic moment value of a small-sized permanent magnet in the susceptometer is $m_{\mathrm{d}} = 0.1$ A·m^2, whereas vertical component of the Earth's magnetic field induction equals $B_{\mathrm{EZ}} = 45$ μT. Let the distance from the magnet to the weight base be$Z_0 = 20$ mm and, previously calculated, effective height of cylindrical equivalent of real 50 g weight $h = 24.7$ mm.

For the numerical values of parameters specified above, within 1-cylinder approximation (recommended by OIML R 111-1 directive) the quantities present in main formulae (1-2) take the following values: $F_{max} = 0.00117$ N, $Z_1 = 44.7$ mm, $I_a = 0.382$, $I_b = 0.858$. As a result the values of magnetic susceptibility and remnant magnetic polarization are:

$$\chi = 0.007336 \qquad \text{and} \qquad \mu_0 M_Z = 5.684\ \mu\text{T}.$$

When considering 3-cylinder magnetic model extension, the numerical values of the quantities present in formulae (5-6) are following:

$$Z_{0I} = Z_0 = 20.0\ \text{mm}, \qquad Z_{1I} = Z_{0I} + h_I = 41.3\ \text{mm},$$

$$Z_{0II} = Z_0 + h_I = 41.3\ \text{mm}, \qquad Z_{1II} = Z_{0II} + h_{II} = 43.3\ \text{mm},$$

$$Z_{0III} = Z_0 + h_I + h_{II} = 43.3\ \text{mm}, \qquad Z_{1III} = Z_{0III} + h_{III} = 46.3\ \text{mm},$$

and

$$I_{aI} = 0.3797, \qquad I_{bI} = 0.8300, \qquad F_{maxI} = 1.17 \cdot 10^{-3}\ \text{N},$$

$$I_{aII} = 0.0100, \qquad I_{bII} = 0.0126, \qquad F_{maxII} = 6.45 \cdot 10^{-5}\ \text{N},$$

$$I_{aIII} = 0.0321, \qquad I_{bIII} = 0.0416, \qquad F_{maxIII} = 5.34 \cdot 10^{-5}\ \text{N}.$$

The estimations of magnetic susceptibility and permanent magnetic polarization (within 3-cylinder model) calculated with formulae (5) and (6) are given below:

$$\chi_{corr} = 0.007343 \qquad \text{and} \qquad \mu_0 M_{Zcorr} = 5.700\ \mu\text{T}.$$

The obtained results are outwardly astonishing, this is because 3-cylinder model correction of magnetic susceptibility is negligible (less than 0.1 %), the same situation takes place in case of the permanent magnetic polarization (less than 0.3 %). Undoubtedly, these outcomes prove, that recommendations included in OIML R 111-1 directive are very effective and sufficiently precise in the most typical weights. It is clear, that two upper cylinders in the 3-cylinder model are of relatively small volume and, their distance from the magnet is greater than the distance of the base cylinder. Thus, they are weakly magnetized and they exert much weaker force on the magnet. However, one can expect entirely different situation in the case of weights with adjusting cavity. The main part of this empty space is located in the base cylinder of weight, thus it should have a considerable impact on the effective magnetic force sensed by the permanent magnet in the susceptometer.

Thus, as a final goal, we intend to apply both analytical n-cylinder model with negative, effective magnetic susceptibility in adjusting cavity region, and the finite element method (FEM). All these tasks will be realized by means of open-source software packages (SMath and FEMM). Moreover, n-cylinder model is to be also implemented as an Excel spreadsheet for easy dissemination of the algorithm.

CENTRUM METROLOGII BADAŃ I CERTYFIKACJI

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26-600 RADOM, ul. Toruńska 5

tel.: +48 (48) 386 60 00, Fax: +48 (48) 385 00 10

www.radwag.com