

Uncertainty of mass measurement in practice.

Each kind of measurement has some uncertainty of results. When providing a physical measuring result, one should also provide data regarding quantity and resolution of such measurements. It is indispensable for anyone who utilizes those measurements in their work, as they also need to settle the reliability of the measurements. If such data are missing, there are no means to compare the measurements and their references mentioned in regulations and norms. So, calculating and expressing the uncertainty is obligatory for a measuring process.

The notion of uncertainty, as a digitally expressed feature is a fairly new way phenomenon in history of measurements, even though, the error and analysis of the errors are present in metrology for very long time.

As calculation of all known and anticipated errors is completed, and the corrections are done, what still remains is the uncertainty regarding the means of obtaining the result and whether this result correctly represents the measurand.

The ideal method of estimating and expressing the uncertainty of measurement should be as universal as possible, in order to be applicable to any kind of measurement and any kind of data processed in measurements.

The value of uncertainty is often expressed with special attention paid to range within the measurements limits, which also covers the distribution of calculated data. This way the grounds for proper calculation of the measured data are specified.

The uncertainty is inseparably bound to measuring results. Recent years have taken special attention to this phenomenon, in standard laboratory and industrial conditions, as well as while providing other analysis (like supply control in production process).

There comes a question of what the uncertainty is. According to International Dictionary of Basic and General Terms of Metrology, the uncertainty of measurement is a parameter bound to the measuring result, which is characterized by dispersion of measurand, and which is an attribute of measurand.

A parameter involving uncertainty is for instance standard deviation or is multiplication. Standard deviation from a series of measurements is also an uncertainty. At that point we come up to a division of uncertainty by the source of parameters. One distinguishes here two

kinds of uncertainty: type A and type B. Further on we shall distinguish the other division of uncertainty into complex and extended.

Uncertainty type A

Method A describes the calculation of standard uncertainty by analyzing the statistic series of observations. In such case the standard uncertainty is actually the standard deviation. This method requires large amount of measurements and their repetitions, and it is successfully implemented in case of random errors. Method A is applied if it is possible to perform a series of equal measurements in equal measuring conditions. Such is the case for checking the repeatability of an electronic balance, where a series of approximately 10 measurements is performed (according to European Union regulation PN-EN 45501:1992 it is a point close to maximal capacity of an electronic balance). It is very important to perform the measurements with the same standard mass, by the same operator, and in shortest possible period of time, with stable environmental conditions. As the measurements are completed, it is possible to calculate standard deviation by:

$$s(x) = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2}$$

where:

n – quantity of repetitions (measurements)

x_i – result i of the measurement,

\bar{x} – average value of all measurements for n repetitions, calculated according to below relation:

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$

Both relations are well known in mathematics, and they are commonly used for measurement analysis. For the uncertainty type A the standard distribution is employed, which is graphically expressed as Gaussian curve. For extremely large quantities of measurements (e.g. $n = 400$), it is possible to experimentally set such curve ourselves. It is a very good example used for training new personnel and comprehending the notion of a measurement.

Uncertainty generally involves a series of aspects. Some of them can be determined by statistic distribution of results in a series of measurements, and they are conceived as standard deviation, as specified above. The other components of uncertainty, even though they are described by standard deviation, are defined according to assumed distribution of probability, which are based on experiments or other data. Here one comes to the second way of determining uncertainty – type B.

Uncertainty type B

Uncertainty type B is determined by a scientific analysis based on all accessible information on changeability of initial value. Those data are: based on previously performed measurements, operator's experience, characteristic features of measured materials and measuring devices. Additionally, method B utilizes data from manufacturer's product specification, uncertainty reference data, handbook and manual content, all accessible publications and other. Data obtained from calibration certificate of a measuring device, standard masses and other certificates are also very important.

With application of a previously mentioned electronic balance, it is possible to determine the constituents of uncertainty type B, which are:

- reading unit d ,
- repeatability, which is determined by standard deviation set earlier by an operator or during calibration process,
- balance indication error, specified in calibration certificate,
- uncertainty while determining an indication error.

On further analysis, one can focus on many other parameters, but depending on the accuracy of a measurement, they may have no influence on the uncertainty value. In case where uncertainty is determined by method B, most common distribution is rectangle shaped. Thus, in order to calculate the value of uncertainty, the initial data should be divided by $\sqrt{3}$. In case of resolution of a measuring device, where one is able to set the upper and lower limit of initial value, the uncertainty is determined by dividing the reading unit by $2\sqrt{3}$. The uncertainty of determining the indication error is calculated by dividing the enhanced uncertainty (specified on the calibration certificate) by enhanced ratio k , which is also

specified on the calibration certificate. Above calculations have led up to further definitions, namely complex uncertainty and extended uncertainty.

Complex uncertainty

Complex uncertainty – in simple words – is a connection of uncertainty type A and type B. the most common is the complex uncertainty, there are, however, some cases, where complete uncertainty analysis is based on the type B.

One of the parameters is so called sensitivity ratio, which is related to the initial rate. It is a partial derivative, which describes how the estimator of initial quantity changes in relation to changes in values of initial estimators. This parameter is characterized by a relation:

$$c_i = \frac{\partial f}{\partial x_i} = \frac{\partial f}{\partial X_i} \Big|_{x_1=x_1 \dots x_N=x_N}$$

where:

c_i – sensitivity ratio

x_i – initial rate estimator

X_i – initial rate value

Participation in the complex uncertainty is expressed by a ratio:

$$u_i(y) = c_i \cdot u(x_i)$$

Where:

$u_i(y)$ – participation in standard complex uncertainty

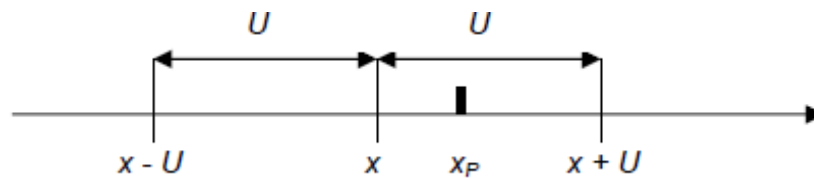
c_i – sensitivity error

$u(x_i)$ – standard uncertainty

Extended uncertainty

Extended uncertainty is a value describing the range of values surrounding the measuring result, which, as expected, can cover a large part of values distribution, which are commonly assigned to measured. According to Guide to Expression of Uncertainty in Measurements, letter u has been assigned to match uncertainty, and expression of extended uncertainty is

realized by capital letter U . Graphic presentation of measurement uncertainty is shown on below chart:



where:

x – measurement result

x_p – measurand

As result of value measurement x_p , value x has been obtained x . As seen above, the result of measurement is not equal to measurand - there are no ideal measurement in the environment. One can only discuss the range in which the measurand is positioned. Depending on the accuracy of a measuring process and the uncertainty related to it, the range can have bigger or smaller scope. Scope size depends for instance on applied measuring device, environmental conditions, operator, and also proper analysis of measuring uncertainty.

An extension ratio k is a numerically expressed ratio, used as a multiplier of standard complex uncertainty, determined to set extended uncertainty.

The extended uncertainty is expressed by a below relationship:

$$U = k \cdot u(x)$$

Where:

U – extended uncertainty

k – extension ratio

$u(x)$ – complex uncertainty

In cases where the distribution of measurand is characterized by a standard (normal) distribution (Gaussian), and the standard uncertainty is related to initial value estimator, the extension ratio k is described as $k = 2$. Such assignment of extended measurement uncertainty, correctly refers to trust level, which equals approximately 95 %. Above case is

correct in vast majority of calibration processes. For this reason international organizations have decided that laboratories which perform calibration and which are accredited as EAL members, should specify the extended uncertainty U , obtained by multiplication of standard uncertainty $u(y)$, initial value estimator y by extension ratio $k = 2$.

When discussing the uncertainty of measurement, one should remember, that it is an effect of random errors, which occur in measuring process. Error of measurement, as described by International Dictionary of Basic and General Terms of Metrology, it is a difference between measuring result and a real measured value.

Following above definition, one can determine below errors:

- relative error, a ratio between the measurement error and real value of measurand;
- random error, which is a difference between measurement value, and an average from infinite quantity of measurements of the same measurand, performed in repeatable conditions.
- systematic error, expressed as a difference between the averages from the infinite quantity of measurements of the same measurand. The notion of systematic error refers to the correction process, which is a added value to the sole measuring result. The correction is to compensate the systematic error, and it is easily presented as a measuring error with reverse value.

Below please find an instance of calculating uncertainty of measurement for 5 gram sample, that is measured on an electronic balance with readability 0,01 mg.

According to the regulations on estimating uncertainty of measurement, the very first step is determining the measurement equation, which should include all elements that may influence the measurement:

The instance we are discussing should be calculated by below equation:

$$m = m_0 + \delta m_1 + \delta m_2 + \delta m_3 + \delta m_4$$

where:

m – measured mass

m_0 – weighed mass

δm_1 – repeatability of balance indications

δm_2 – balance reading unit

δm_3 – balance indication error

δm_4 – uncertainty on determining indication error.

As the equation is properly prepared, one should focus on determining the equation for uncertainty of measurement, which calculates uncertainty for each element of the equation:

$$u_2(m) = u_2(\delta m_1) + u_2(\delta m_2) + u_2(\delta m_3) + u_2(\delta m_4)$$

$$c_i = 1$$

The sensitivity ratio in above case equals 1 for each element of the equation. The next step determines calculation of uncertainty for initial value of each element:

– weighed mass – m_0 : as the sample is placed on the weighing pan of a balance, its mass is displayed as 5000 mg (for the purpose of simplification all below masses will be presented as mg),

– repeatability of balance indications – δm_1 : based on several series of measurements, standard deviation equaling $s = 0,02$ mg has been determined,

– balance reading unit – δm_2 : reading unit δ of the applied balance equals 0,01 mg, and thus the uncertainty referring to the resolution of the measuring device should equal:

$$u(m_2) = \frac{0,01\text{mg}}{2\sqrt{3}} = 0,0029\text{mg}$$

– balance indication error – δm_3 : the calibration certificate of the balance gives indication error of + 0,01 mg for 5 g, with uncertainty of measurement equal $U = 0,02$ mg and extension ratio equal $k = 2$. The uncertainty is calculated with equation:

$$u(m_3) = \frac{0,01\text{mg}}{\sqrt{3}} = 0,0058\text{mg}$$

$$u(m_4) = \frac{0,02\text{mg}}{2} = 0,01\text{mg}$$

The next step is collecting all above results and formulating uncertainty budget, thanks to which it is possible to observe, which of the elements influences the uncertainty the most (chart 1). Uncertainty value is determined as a radical sum of squares of all uncertainty elements (contribution into complex uncertainty).

Symbol value	Value estimator	Standard uncertainty	Probability distribution	Sensitivity ratio	Contribution into complex uncertainty
m 0	5000 mg	-	-	-	-
δ m1	0 mg	0,0200 mg	<i>standard</i>	1	0,0200 mg
δ m2	0 mg	0,0290 mg	<i>rectangular</i>	1	0,02900 mg
δ m3	0 mg	0,058 mg	<i>rectangular</i>	1	0,05800 mg
δ m4	0 mg	0,0100 mg	<i>standard</i>	1	0,0100 mg
m	5000 mg			Uncertainty	0,023 mg

Chart 11. An instance of uncertainty budget.

In accordance to the procedure, the next step is calculating extended uncertainty U . it has been assumed in the above case, that the extension ratio equals $k = 2$, which corresponds to trust level of approximately 95%.

If the relationship describing extended uncertainty is applied, than the earlier mentioned value of extended uncertainty is calculated with a equation:

$$U = k \cdot u_c(m) = 2 \cdot 0,023\text{mg} = 0,046\text{mg}$$

The final result of the measurement, that is the indication of the balance with a 5 gram load on its weighing pan equals

$$m = (5000,00 \pm 0,05) \text{ mg}$$

and so, the measurand is within the thresholds from 4999,95 mg to 5000,05 mg.

The uncertainty of a measuring result expressed the lack of accurate data on quantity of measured value. The accurate knowledge on quantity of measured value requires infinite quantity of information, and thus in practice is unobtainable. Phenomena affecting the uncertainty, and thus the fact that the measurement result cannot be expressed using one value, is the source of uncertainty. In practice, we can identify several possible sources of uncertainty, which are, inter alia:

- incomplete definition of a measurand,
- imperfect realization of the definition of the measurand,
- imprecise sampling, ie, the measured sample is not representative for defined measured quantity,
- Incomplete knowledge of the impact of environmental conditions on the measurement procedure, or imperfect measurement of the parameters characterizing these conditions,
- Subjective errors in reading the indications of analog instruments,
- Inaccurately known values assigned to standard masses and reference materials,
- Inaccurately known values of constant and other parameters, obtained from external resources, and applied in data processing procedures,
- Simplifying approximations and assumptions used in methods and measuring procedures,
- Dispersion of values obtained in the process of observations repeated in seemingly equal conditions.

The success in estimating uncertainty of measurement depends on thorough and correct analysis of the whole measuring process. It is very important, that the estimation of measurement uncertainty was appropriate to its accuracy, since not always the components of the uncertainty are influential to the process itself.